Effects of electron-electron interaction on the conductance of open quantum dots

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We study the effect of electron-electron interaction on the conductance of open quantum dots. We find that Coulomb interactions (i) do not affect the ensemble averaged conductance $\langle G\rangle$ if time-reversal symmetry has been broken by a magnetic field, (ii) enhance weak localization and weak anti-localization corrections to $\langle G\rangle$ in the absence of a magnetic field, (iii) increase conductance fluctuations, and (iv) enhance the effect of short trajectories on the conductivity of quantum dot.

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The phenomenon of Coulomb blockade in quantum dots is commonly associated with dots that are coupled to the outside world via tunneling contacts [1]. In these systems, the total charge on the dot is quantized in units of the electron charge e. Once the temperature T becomes smaller than the charging energy E_C (the energy needed to add an extra electron to the system), transport through the quantum dot is suppressed, unless the system is tuned to a charge degeneracy point. On the other hand, if the connection to the outside world is via ballistic point contacts with a conductance much larger than the conductance quantum $2e^2/h$, charge is no longer quantized, and the Coulomb blockade is lifted.

In recent years, it has become possible to study the intermediate regime, of quantum dots with ballistic point contacts with only a few propagating channels at the Fermi level, so that their conductance is $\sim e^2/h$. For temperatures T comparable to the spacing Δ of single-particle levels in the quantum dot, the conductance of these open quantum dots shows mesoscopic fluctuations of the same order as the average [2,3]. Theoretically, the large mesoscopic fluctuations are understood within the framework of random-matrix theory [4–7], which employs a picture of non-interacting electrons, and does not account for Coulomb blockade effects. A partial justification for this approach can be obtained from the work of Furusaki and Matveev [8], who calculated the effect of electron-electron interactions on the conductance G of a quantum dot with ballistic single-channel point contacts in the limit $E_C \gg T \gg \Delta$, where there are no mesoscopic fluctuations. They found $G = e^2/h$, indicating that there is no Coulomb blockade effect on the conductance for ballistic point contacts in this limit. At the same time, the presence of weakly reflecting tunnel barrier in the contact will drive the system into a state of Coulomb blockade, in which transport is inhibited.

That this is not the complete picture was explained in a recent work by one of the authors and Glazman [9], who showed that as a result of the interplay of mesoscopic fluctuations and electron-electron interactions, Coulomb blockade effects on the capacitance and the tunneling density of states of a quantum dot persist, even at a perfect transparency of the point contacts. This so-called "mesoscopic charge quantization" urges us to reconsider the effect of Coulomb interactions on the conductance of a quantum dot with ideal point contacts. In this letter we report a calculation the conductance of such a quantum dot and its mesoscopic fluctuations, in the presence of electron-electron interactions.

We consider a quantum dot connected to electron reservoirs via two leads (labeled 1 and 2), see Fig. 1. In each of the leads there are N propagating channels at the Fermi level E_F. For the Hamiltonian of the system we take

$$\mathcal{H} = \mathcal{H}_F + \mathcal{H}_C, \tag{1a}$$

where $\mathcal{H}_F$ is the Hamiltonian of the non-interacting electrons and $\mathcal{H}_C$ is the interaction Hamiltonian, corresponding to a capacitive interaction inside the quantum dot,

$$\mathcal{H}_F = \sum_{\sigma=\pm} \int d\vec{r}\, \psi_\sigma^\dagger \left(-\frac{1}{2m}\nabla^2 + U(\vec{r}) - \mu\right) \psi_\sigma, \tag{1b}$$

$$\mathcal{H}_C = \frac{E_C}{2e^2}\left(Q - ne\right)^2, \quad Q = e \sum_{\sigma=\pm} \int_{x>0} d\vec{r}\, \psi_\sigma^\dagger \psi_\sigma. \tag{1c}$$

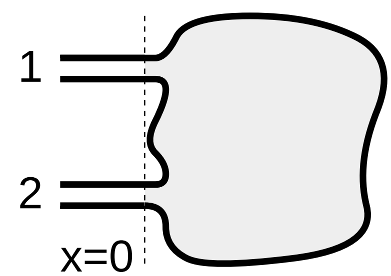

FIG. 1. Quantum dot (dotted) connected to two ideal leads (labeled 1 and 2).

The index σ denotes spin. The potential $U(\vec{r})$ describes the confinement by the gates surrounding the quantum dot and the leads. The leads are located at $x < 0$, and the dot at $x > 0$, see Fig. 1. As is explained in Ref. [9], the interaction Hamiltonian $\mathcal{H}_C$ with the capacitive interaction

provides a sufficient description of the electron-electron interactions in a metal of semiconductor quantum dot if the electrons explore the dot ergodically before they exit the dot through one of the point contacts, i.e. if all the involved energy scales are much smaller than the Thouless energy of the dot E_T. It is in this regime that the conductance statistics become universal and random-matrix theory can be used as a model for the non-interacting Hamiltonian $\mathcal{H}_F$ [7].

To summarize the result of our calculation, which is described below, we find that the presence of the interaction Hamiltonian (1c) enhances mesoscopic (quantum-interference) corrections to the conductance. In particular:

(i). There is no interaction correction for the ensemble averaged conductance $\langle G\rangle$ in the presence of a time-reversal symmetry breaking field (unitary ensemble). This is different from disordered bulk systems, where interactions do have an effect on the conductance, even in the presence of a weak magnetic field [10].

(ii). In the absence of a magnetic field (orthogonal ensemble), the weak-localization correction to $\langle G\rangle$ is enhanced by the interactions. We have computed $\langle G\rangle$ in a perturbation series in $N\Delta/T$, where N is the number of channels in each of the point contacts and Δ is the mean level spacing in the quantum dot,

$$\langle G\rangle = \frac{N}{2} - \frac{N}{4N+2} - \frac{c_N}{N}\frac{N\Delta}{8\pi^2 T}. \tag{2}$$

Here c_N is a numerical constant ranging from $c_1 \approx 3.18$ to $c_\infty = \pi^2/6$. The conductance is measured in units of $2e^2/h$. The second term in Eq. (2) is the usual weak-localization correction for non-interacting particles [4], the third term is the interaction correction. Within the model (1) there is no dephasing, which would give rise to a suppression of weak localization. The origin of the absence of dephasing is that all interference occurs inside the dot, and the interaction Hamiltonian does not produce excitations during the course of such motion. In the presence of spin-orbit scattering (symplectic ensemble), the corrections to the conductance [second and third term on the r.h.s. of Eq. (2)] are multiplied by $-1/2$. Hence, in this case, Coulomb interactions enhance the conductance. We expect that the conductance distribution saturates when πT becomes comparable to the level broadening $N\Delta/\pi$. Hence for low temperatures, the interaction correction to $\langle G\rangle$ remains small as $1/N$, and there is no transition to the regime of real Coulomb blockade.

(iii). The capacitive interaction in the Hamiltonian (1) enhances the mesoscopic conductance fluctuations. In the unitary ensemble, the conductance G shows sample-to-sample fluctuations with variance

$$\operatorname{var} G = \frac{N\Delta}{96T} + \frac{c_N}{N}\frac{N^2\Delta^2}{32\pi^2 T^2} \quad \text{for } T \gg N\Delta, \tag{3}$$

with $c_N \approx 6.49$ for $N \gg 1$. In the absence of a magnetic field, the fluctuations are larger by a factor 2, while the effect of strong spin-orbit scattering is to reduce $\operatorname{var} G$ by a factor 4. The first term on the r.h.s. of Eq. (3), which represents the conductance fluctuations for non-interacting electrons, is derived in the limit of large channel numbers, $N \gg 1$, and agrees with the result obtained by Efetov [6]. The second term is the interaction correction.

(iv). The above results are for ideal ballistic point contacts and for ergodic quantum dots. The presence of a weakly scattering impurity in the point contact with reflection amplitude r gives rise to a correction to the average conductance that behaves as

$$\delta G = -\frac{N^2}{2\pi} c_N |r|^2 \left(\frac{2NE_C\gamma}{\pi^2 T}\right)^{1/2N} \sin\frac{\pi}{4N}, \tag{4}$$

where $c_1 \approx 5.32$ and $c_N \to 4$ for $N \gg 1$. For $N = 1$ this result has been obtained previously in Ref. [8]. However, not only an impurity in the contact, but also any other direct scattering process that acts on a time scale shorter than the Coulomb time $t_c \propto 1/E_C$ (like scattering from short trajectories connecting the point contacts), has the same effect on the conductance. The effect of all direct processes is summarized by the replacement of the reflection probability $|r|^2$ by the difference $|r|^2 - |t|^2$ of the probabilities for direct reflection and transmission, respectively. In particular, we expect that as a result of interactions, the conductance of an open quantum dot with a direct path connecting source and drain contacts *increases* as the temperature T is lowered.

Let us now proceed with the formulation of the theoretical framework used for the derivation of these results. The conductance is computed from the current-current correlator in imaginary time,

$$G = \frac{1}{2T}\int_{-\infty}^{\infty} dt\, \Pi\left(it + \frac{1}{2T}\right), \tag{5a}$$

$$\Pi(\tau) = \langle T_\tau I(\tau) I(0)\rangle. \tag{5b}$$

We linearize the spectrum in the leads. In each lead there are N channels, labeled $j = 1, \ldots, N$ for lead 1 and $j = N+1, \ldots, 2N$ for lead 2. The current I is defined in the leads, just outside the quantum dot,

$$I = \frac{e}{4\pi}\sum_{j=1}^{2N}\sum_{\sigma=\pm}\nu(j) \times \left(\psi^\dagger_{Lj\sigma}(x)\psi_{Lj\sigma}(x) - \psi^\dagger_{Rj\sigma}(x)\psi_{Rj\sigma}(x)\right), \tag{6}$$

where $\psi^\dagger_{Lj\sigma}$ ($\psi^\dagger_{Rj\sigma}$) and $\psi_{Lj\sigma}$ ($\psi_{Rj\sigma}$) denote creation and annihilation operators for left (right) moving particles with spin σ in channel j, and $\nu(j) = 1$ if $1 \le j \le N$ and -1 else. (We have chosen units such that the Fermi velocity $v_F = 1/2\pi$.) We take the limit $x \uparrow 0$ at the end of the calculation.

The dynamics of the quantum dot is dealt with in the effective action method of Ref. [9]. Following Ref. [9], the charge Q of the quantum dot in the interaction Hamiltonian $\mathcal{H}_C$ is replaced by the charge Q_L of the leads. Such a replacement is allowed, because the total number of particles (in the dot and in the leads) is conserved. Next, the degrees of freedom of the quantum dot are integrated out, in favor of a "mirror" copy of the leads at $x > 0$, and an effective action $\mathcal{S}_{\text{eff}}$, to be defined below. The Hamiltonian of the system now reads

$$\mathcal{H} = \frac{i}{2\pi}\sum_{j=1}^{2N}\sum_{\sigma=\pm}\int_{-\infty}^{\infty} dx \left(\psi^\dagger_{Lj\sigma}\partial_x\psi_{Lj\sigma} - \psi^\dagger_{Rj\sigma}\partial_x\psi_{Rj\sigma}\right) + \frac{E_C}{2e^2}(Q_L + ne)^2, \tag{7a}$$

$$Q_L = e\sum_{j=1}^{2N}\sum_{\sigma=\pm}\int_{-\infty}^{0} dx \, :\psi^\dagger_{Lj\sigma}\psi_{Lj\sigma} + \psi^\dagger_{Rj}\psi_{Rj\sigma}: . \tag{7b}$$

The scattering of particles from the quantum dot is represented by the imaginary-time effective action $\mathcal{S}_{\text{eff}}$ acting at $x = 0$,

$$\mathcal{S}_{\text{eff}} = \sum_{i,j,\sigma,\sigma'}\int_0^{1/T} d\tau_1\, d\tau_2\, [\bar\psi_{Li\sigma}(\tau_1;0) + \bar\psi_{Ri\sigma}(\tau_1;0)] \times L_{ij;\sigma\sigma'}(\tau_1-\tau_2)\,[\psi_{Lj\sigma'}(\tau_2;0) + \psi_{Rj\sigma'}(\tau_2;0)],$$

where $\psi(\tau;x) = e^{\mathcal{H}\tau}\psi(0;x)e^{-\mathcal{H}\tau}$ and $\bar\psi(\tau;x) = \psi^\dagger(-\tau;x)$. The kernel $L_{ij;\sigma\sigma'}(\tau)$ is a hermitian matrix, related to the scattering matrix $S_{ij;\sigma,\sigma'}$ of the quantum dot [9],

$$L(\omega_n) = \int_0^{1/T} e^{i\omega_n\tau}L(\tau)d\tau = \frac{1}{4\pi i}\frac{1-S(\omega_n)}{1+S(\omega_n)} - \frac{1}{4\pi i}\,\text{sgn}(\omega_n), \tag{8}$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara frequency. Note that in Matsubara representation, the kernel $L(\omega_n)$ satisfies $L(\omega_n) = L(-\omega_n)^\dagger$. The current correlator $\Pi(\tau)$ is now calculated as a thermal average with respect to the Hamiltonian $\mathcal{H}$ and with the effective action $\mathcal{S}_{\text{eff}}$,

$$\Pi(\tau) = \frac{\langle T_\tau I(\tau)I(0)e^{\mathcal{S}_{\text{eff}}}\rangle_{\mathcal{H}}}{\langle e^{\mathcal{S}_{\text{eff}}}\rangle_{\mathcal{H}}}, \tag{9}$$

To compute the thermal average of the interacting system defined by Eqs. (5)–(9), we first bosonize the one-dimensional Hamiltonian $\mathcal{H}$ of Eq. (7). In this way the interaction becomes quadratic in terms of the boson fields, and can be dealt with exactly. To account for the scattering from the quantum dot, which is represented by the effective action $\mathcal{S}_{\text{eff}}$, we perform an expansion up to second order in powers of the scattering matrix S. To be precise, we first expand $\Pi(\tau)$ in powers of the action $\mathcal{S}_{\text{eff}}$, and then we expand $\mathcal{S}_{\text{eff}}$ in powers of the scattering matrix S using that up to order S^2 we have $2\pi i L(\omega_n) = -S(\omega_n)$ if $\omega_n > 0$ and $S^\dagger(-\omega_n)$ otherwise, see Eq. (8). Note that in Ref. [9] the kernel L was used as the expansion parameter. The expansion in the scattering matrix S instead of the kernel L guarantees that in the absence of interactions mesoscopic fluctuations are fully accounted for. Only corrections to the conductance that depend on the interplay of interactions and mesoscopic fluctuations are taken into account perturbatively. The perturbation expansion is valid in the regime $T \gg N\Delta$. We find in this way

$$G = \frac{N}{2} - \frac{\pi T}{8}\int_0^\infty dt_1\, dt_2 \sum_{k,l,\sigma,\sigma'} S^*_{lk;\sigma\sigma'}(t_1)S_{lk;\sigma\sigma'}(t_2)\nu(k)\nu(l)\left\{\frac{t_2-t_1}{\sinh[(t_2-t_1)\pi T]} + T\sin\frac{\pi}{4N} \times\int_{t_c}^\infty ds\frac{2s+t_2+t_1}{\sinh[(s+t_2)\pi T]\sinh[(s+t_1)\pi T]}\left(\frac{\sinh[(t_2+t_1+s+t_c)\pi T]\sinh[(s-t_c)\pi T]}{\sinh[(t_1+t_c)\pi T]\sinh[(t_2+t_c)\pi T]}\right)^{1/4N}\right\}. \tag{10}$$

Here $t_c = \pi/2NE_C\gamma$, γ being the Euler constant. The real-time or Lehmann representation of the scattering matrix is defined through $S(\omega_n) = \int_0^\infty S(t)e^{-\omega_n t}$, $\omega_n > 0$.

Equation (10) contains the conductance of a specific sample in the presence of the interaction (1c), up to second order in the scattering matrix $S(t)$ of the quantum dot. The first term between brackets is nothing else than the Landauer formula for the conductance of the non-interacting system. The remaining term is the interaction correction. The perturbation theory in powers of the scattering matrix is arranged in such a way, that we handle the non-interacting "pole" contributions exactly, and do a perturbation theory in the interacting "branch-cut" terms. The sample-specific conductance (10) was obtained by a *thermal* average of the interacting system. Next, we need to take a *mesoscopic* average over an ensemble of quantum dots. Such an ensemble can be obtained by varying e.g. the shape, impurity configuration, or the Fermi energy of the quantum dot. The correlators of the scattering matrix for such a mesoscopic ensemble have been studied extensively in the literature [7]. Although the two-point correlator of scattering matrix elements is known exactly [11], for our purposes, within the regime $T \gg N\Delta$ where the expansion (10) is valid, it is sufficient to use the asymptotic large-N formulas for these correlators. In the absence of spin-orbit scattering, the scattering matrix is diagonal in spin space, $S_{ab;\sigma\sigma'} = S_{ab}\delta_{\sigma\sigma'}$. For the unitary ensemble (presence of a time-reversal symmetry breaking magnetic field), we have

$$\langle S^*_{ab}(t) S_{a'b'}(t') \rangle = \delta_{aa'}\delta_{bb'} \frac{\Delta}{2\pi} e^{-N\Delta t/\pi} \delta(t'-t), \tag{11a}$$

while in the presence of time-reversal symmetry (orthogonal ensemble) we find

$$\langle S^*_{ab}(t) S_{a'b'}(t') \rangle = (\delta_{aa'}\delta_{bb'} + \delta_{ab'}\delta_{ba'}) \frac{\Delta}{2\pi} e^{-(2N+1)\Delta t/2\pi} \delta(t'-t). \tag{11b}$$

With spin orbit scattering in the presence of time-reversal symmetry (symplectic ensemble), we have

$$\langle S^*_{ab;\sigma\tau}(t) S_{a'b';\sigma'\tau'}(t') \rangle = (\delta_{aa'}\delta_{bb'}\delta_{\sigma\sigma'}\delta_{\tau\tau'} - \delta_{ab'}\delta_{ba'}(\sigma_y)_{\sigma\tau'}(\sigma_y)_{\tau\sigma'}) \frac{\Delta}{2\pi} e^{-(4N-1)\Delta t/4\pi} \delta(t'-t), \tag{11c}$$

where σ_y is the Pauli matrix. For interaction corrections to leading order in $N\Delta/T$, higher order correlators of S can be taken Gaussian. The ensemble average of Eq. (10) with the help of the correlators (11) yields the weak-localization correction (2) and the conductance fluctuations (3). (The observation that $\langle G \rangle = N/2$ in the unitary ensemble follows from symmetry properties of the ensemble of scattering matrices, and holds to arbitrary order in perturbation theory.) The theory of Ref. [8], where the effect of weakly reflecting impurities in the contacts was considered, is recovered by setting $S_{ij;\sigma\sigma'}(t) = r_j \delta(t)\delta_{ij}\delta_{\sigma\sigma'}$, r_j being the reflection amplitude in channel j.

For spinless particles (which might e.g. be realized in spin-polarized quantum dots), Eq. (10) remains valid, provided one substitutes $N \to N/2$. However, if the leads contain only one channel ($N = 1$), an extra n-dependent term δG, has to be added,

$$\delta G = \frac{\pi T^2}{4} \int_{t_c}^{\infty} ds \int_0^{\infty} dt_1 dt_2 \frac{\mathrm{Re}\, e^{-2\pi i n}(2s+t_1+t_2)\,[S_{12}(t_1)S_{21}(t_2) - S_{11}(t_1)S_{22}(t_2)]}{\sqrt{\sinh[(t_c+t_1)\pi T]\sinh[(t_c+t_2)\pi T]\sinh[(t_c+s+t_2+t_1)\pi T]\sinh[(s-t_c)\pi T]}}. \tag{12}$$

In this case the weak localization correction to the conductance is enhanced,

$$\langle G \rangle = \frac{1}{6} + \frac{\Delta}{16\pi T} \ln \frac{t_c \pi T}{2c}, \tag{13}$$

where $c \approx 0.36$. Eq. (12) bears explicit reference to the particle number n, and induces an explicit n-dependence in the conductance fluctuations,

$$\langle G(n)G(n') \rangle = \frac{c\Delta^2}{128\,\pi^2 T^2} \cos[2\pi(n-n')] \ln^2(t_c \pi T) + n\text{-independent terms}, \tag{14}$$

where $c = 5/4$ $(1/2)$ in the presence (absence) of time-reversal symmetry. In our perturbation theory, such an explicit n-dependence, which reflects the discreteness of charge, is absent for particles with spin. The origin for the appearance of this extra term for spinless particles and $N = 1$ is the same as in Ref. [8]. For spin 1/2 electrons, effects of the discreteness of charge appear in general in $4N$th order in perturbation theory in S.

We close with a remark on the issue of dephasing, which plays an important role in the experiments on open quantum dots [2]. While the experiments indicate that the dephasing time τ_ϕ diverges as T^{-1} as the temperature $T \to 0$, the theoretical prediction for a closed quantum dot is $\tau_\phi \propto T^{-2}$ [12]. As our model (1), which takes the effect of open contacts into account, does not give rise to significant dephasing effects, we believe that the fact that a quantum dot is open by itself does not lead to additional dephasing, and cannot explain the puzzling discrepancy between experiment and theory.

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